The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

1. The law says that it is a crime for an American to sell weapons to hostile nations.

**∀ x,y,z American(x) AND weapon(y) AND hostile\_nation(z) AND sell(x,y,z) 🡺 criminal(x)**

1. The country Nono

Country(nono)

1. Nono, an enemy of America

Enemy(Nono, America)

1. has some missiles

**∃m missile(m) AND** has(Nono,m)

1. all of its missiles were sold to it by Colonel West

**∀m1 missile(m1) AND has(Nono,m1) 🡺 sell(West, m1, Nono)**

1. Colonel West, who is American

American(West)

1. Prove: Colonel West is a criminal

Criminal(West) ??

Additional knowledge

1. All missiles are weapons

**∀ m2 missile(m2) 🡺 weapon(m2)**

1. America’s enemy countries are hostile

**∀ c country(c ) AND enemy(c, America) 🡺 hostile\_nation(c )**

1. All countries are nations and vice a versa

**∀ c1 country(c1) 🡸🡺 nation(c1)**

**From 4, and AND-Elimination rule**

**11a. missile(m)**

**11b. has(Nono,m)**

**From 11a, 8, modus ponens and substitution m2/m**

1. **Weapon(m)**

**From 2,3,9, modus ponens, generalized substitution, c/Nono**

1. **Hostile\_nation(Nono)**

**From 11a, 11b, modus ponens, substitution, m/m1**

1. **Sells(West,m1,Nono)**

From 6, 12, 13, 14, 1 modus ponens, **generalized substitution x/West, y/m1, z/Nono**

1. Criminal(West)

Hence proved

* 1. Marcus was a man.
  2. Marcus was a Pompeian.
  3. All Pompeian were Romans.
  4. Caesar was a ruler.
  5. All Romans were either loyal to Caesar or hated him.
  6. Everyone is loyal to someone.
  7. People only try to assassinate rulers they aren't loyal to.
  8. Marcus tried to assassinate Caesar.

Prove Marcus is not loyal to Caeser

Represent in FOL

1. man(marcus)
2. Pompeian(Marcus)
3. For all p, Pompeian(p) 🡺 roman(p)
4. Ruler(Caesar)
5. For all r roman(r) 🡺 loyalto(r,Caesar) or hate(r,Caesar)
6. For all x, There exist y loyalto(x,y)
7. For all a,b person(a) AND ruler(b) AND try\_assaninate(a,b) 🡺 ~loyalto(as,b)
8. try\_assaninate(Marcus, Caesar)
9. Prove ~loyalto(Marcus,Caesar)

Additional knowledge

1. All men are people

For all m, man(m) 🡺 person(m)

**sell(x,y,z)**

**sell(West,m,Nono)**

**x/West**

**y/m**

z/Nono

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 4 | **B** |  | **B** | **Pit** |
| 3 | **Pit** | **B Gold** | **Pit** | **B** |
| 2 | **B** |  | **S , B** |  |
| 1 | **Agent** | **S** | **Wumpus** | **S** |
|  | 1 | 2 | 3 | 4 |

Prove: Cell [2,2] doesn’t have a pit using FOL using forward chaining.

Given:

1. Wherever there is a pit, adjacent cells have breeze and vice a versa.

Ɐx, y Breeze[x,y] 🡸🡺 Pit[x+1,y] or Pit[x-1,y] or Pit[x,y-1] or Pit[x,y+1]

1. Wherever there is a Wumpus, adjacent cells have stench and vice a versa

Ɐx, y Stench[x,y] 🡸🡺 Wumpus[x+1,y] or Wumpus [x-1,y] or Wumpus [x,y-1] or Wumpus [x,y+1]

1. There is no pit and there is no wumpus in [1,1]

~Pit[1,1] AND ~Wumpus[1,1]

**Percept:**

1. **~Breeze[1,1]**
2. **~Stench[1,1]**

//**Prove first [1,2] and [2,1] are safe i.e. no pit or Wumpus in [1,2] and [2,1]**

From 1 and bidirectional elimination,

1. Ɐx, y { Breeze[x,y] 🡺 Pit[x+1,y] or Pit[x-1,y] or Pit[x,y-1] or Pit[x,y+1] } AND

{Pit[x+1,y] or Pit[x-1,y] or Pit[x,y-1] or Pit[x,y+1 or Pit[x+1,y]] 🡺 Breeze[x,y]}

From 6 and And-elimination

1. A. Breeze[x,y] 🡺 Pit[x+1,y] or Pit[x-1,y] or Pit[x,y-1] or Pit[x,y+1] or Pit[x+1,y]

B. Pit[x-1,y] or Pit[x,y-1] or Pit[x,y+1] 🡺 Breeze[x,y]

From 7B, Modus tollens(Law of contrapositive)

1. ~breeze(x,y) 🡺 ~( Pit[x-1,y] or Pit[x,y-1] or Pit[x,y+1] or Pit[x+1,y])

From 4, 8, modus ponens, generalized substitution, x/1, y/1

1. ~( Pit[1,2] or Pit[2,1])

Apply negation to every single term in 9,

1. ~ Pit[1,2] AND ~Pit(2,1)

Apply and-elimination to 10

1. A. ~ Pit[1,2]

B. ~Pit(2,1)

1. A. ~Wumpus[1,2]

B. ~Wumpus[2,1]

**Percept:**

**13: there is breeze in [2,1]**

**Breeze[2,1]**

1. **Stench in [1,2]**

**Stench[1,2]**

From 7A, 13, Modus ponens, generalized substitution, x/2, y/1

1. Pit[3,1] or Pit[1,1] or Pit[2,2]

From 3 and And-elimination

1. A. ~Pit[1,1]

B. ~Wumpus[1,1]

From 15, 16A, and law of validity

1. Pit[3,1] or Pit[2,2]

From 2, Bidirectional elimination followed by And-Elimination

1. A. For all x, y Stench[x,y] 🡺 Wumpus[x+1,y] or Wumpus [x-1,y] or Wumpus [x,y-1] or Wumpus [x,y+1]

B. For all x, y Wumpus[x+1,y] or Wumpus [x-1,y] or Wumpus [x,y-1] or Wumpus [x,y+1] 🡺 Stench[x,y]

From 14, 18A, generalized modus ponens, x/1, y/2

1. Wumpus[2,1] or Wumpus [1,1] or Wumpus [1,2]

From 16B, 19, and law of validity

1. Wumpus[2,1] or Wumpus [1,2]

**Percept:**

1. **~Stench[2,1]**
2. **~Breeze[1,2]**

From 8, 22, generalized modus ponens, x/1, y/2

1. ~breeze(1,2) 🡺 ~( Pit[1,1] or Pit[2,2] or Pit[1,3]) 🡺 ~( Pit[1,1] or Pit[2,2] or Pit[1,3])

Applying ~ to every term in 23,

1. ~Pit[1,1] AND ~Pit[2,2] AND ~Pit[1,3]

Applying And-elimination to 24,

1. A. ~Pit[1,1]

**B. ~Pit[2,2]**

C. ~Pit[1,3]

Hence Proved.